

SEVENTH EDITION

CALCULUS

MULTIVARIABLE

Huges-Hallett Gleason McCallum et al.

WILEY

Lines

Slope of line through (x_1, y_1) and (x_2, y_2) :

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-slope equation of line through (x_1, y_1) with slope m :

$$y - y_1 = m(x - x_1)$$

Slope-intercept equation of line with slope m and y -intercept b :

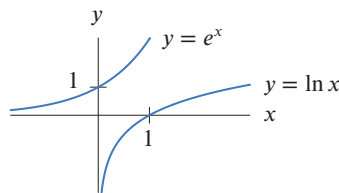
$$y = b + mx$$

Rules of Exponents

$$\begin{aligned} a^x a^t &= a^{x+t} \\ \frac{a^x}{a^t} &= a^{x-t} \\ (a^x)^t &= a^{xt} \end{aligned}$$

Definition of Natural Log

$y = \ln x$ means $e^y = x$
ex: $\ln 1 = 0$ since $e^0 = 1$



Identities

$$\begin{aligned} \ln e^x &= x \\ e^{\ln x} &= x \end{aligned}$$

Rules of Natural Logarithms

$$\begin{aligned} \ln(AB) &= \ln A + \ln B \\ \ln\left(\frac{A}{B}\right) &= \ln A - \ln B \\ \ln A^p &= p \ln A \end{aligned}$$

Distance and Midpoint Formulas

Distance D between (x_1, y_1) and (x_2, y_2) :

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of (x_1, y_1) and (x_2, y_2) :

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Quadratic Formula

If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Factoring Special Polynomials

$$\begin{aligned} x^2 - y^2 &= (x + y)(x - y) \\ x^3 + y^3 &= (x + y)(x^2 - xy + y^2) \\ x^3 - y^3 &= (x - y)(x^2 + xy + y^2) \end{aligned}$$

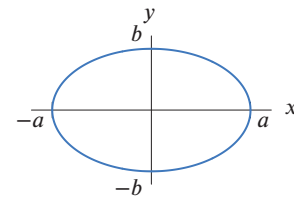
Circles

Center (h, k) and radius r :

$$(x - h)^2 + (y - k)^2 = r^2$$

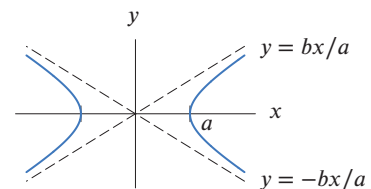
Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



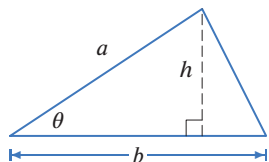
Geometric Formulas

Conversion Between Radians and Degrees: π radians = 180°

Triangle

$$A = \frac{1}{2}bh$$

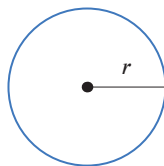
$$= \frac{1}{2}ab \sin \theta$$



Circle

$$A = \pi r^2$$

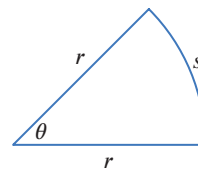
$$C = 2\pi r$$



Sector of Circle

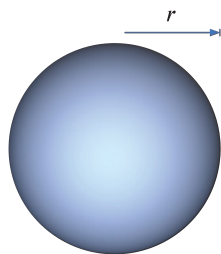
$$A = \frac{1}{2}r^2\theta \quad (\theta \text{ in radians})$$

$$s = r\theta \quad (\theta \text{ in radians})$$



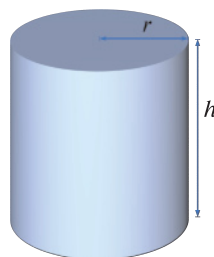
Sphere

$$V = \frac{4}{3}\pi r^3 \quad A = 4\pi r^2$$



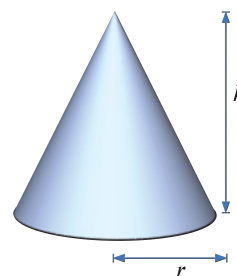
Cylinder

$$V = \pi r^2 h$$



Cone

$$V = \frac{1}{3}\pi r^2 h$$



Trigonometric Functions

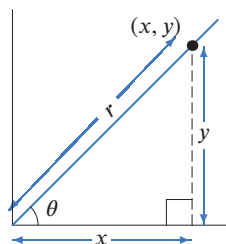
$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

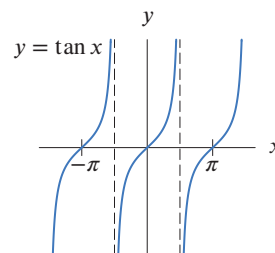
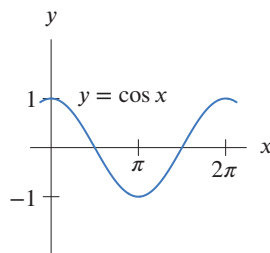
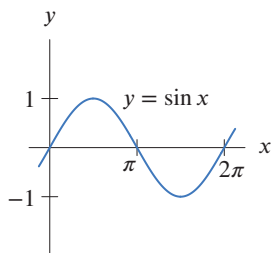


$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin(2A) = 2 \sin A \cos A$$

$$\cos(2A) = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$



The Binomial Theorem

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{1 \cdot 2}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}x^{n-3}y^3 + \dots + nxy^{n-1} + y^n$$

$$(x - y)^n = x^n - nx^{n-1}y + \frac{n(n-1)}{1 \cdot 2}x^{n-2}y^2 - \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}x^{n-3}y^3 + \dots \pm nxy^{n-1} \mp y^n$$

MULTIVARIABLE CALCULUS

Seventh Edition

We dedicate this book to Andrew M. Gleason.

*His brilliance and the extraordinary kindness and
dignity with which he treated others made an
enormous difference to us, and to many, many people.
Andy brought out the best in everyone.*

*Deb Hughes Hallett
for the Calculus Consortium*

MULTIVARIABLE CALCULUS

Seventh Edition

Produced by the Calculus Consortium and initially funded by a National Science Foundation Grant.

William G. McCallum Deborah Hughes-Hallett
University of Arizona *University of Arizona*

Daniel Flath Brad G. Osgood
Macalester College *Stanford University*

Andrew M. Gleason Cody L. Patterson
Harvard University *University of Arizona*

Selin Kalaycıoğlu Douglas Quinney
New York University *University of Keele*

Brigitte Lahme Ayşe Şahin
Sonoma State University *Wright State University*

Patti Frazer Lock Adam H. Spiegel
St. Lawrence University *Loyola University Chicago*

Guadalupe I. Lozano Jeff Tecosky-Feldman
University of Arizona *Haverford College*

Jerry Morris Thomas W. Tucker
Sonoma State University *Colgate University*

David Mumford Aaron D. Wootton
Brown University *University of Portland*

with the assistance of
Adrian Iovita
University of Washington

Coordinated by
Elliot J. Marks

WILEY

ACQUISITIONS EDITOR	Shannon Corliss
VICE PRESIDENT AND DIRECTOR	Laurie Rosatone
DEVELOPMENT EDITOR	Adria Giattino
FREELANCE DEVELOPMENTAL EDITOR	Anne Scanlan-Rohrer/Two Ravens Editorial
MARKETING MANAGER	John LaVacca
SENIOR PRODUCT DESIGNER	David Dietz
SENIOR PRODUCTION EDITOR	Laura Abrams
COVER DESIGNER	Maureen Eide
COVER AND CHAPTER OPENING PHOTO	©Patrick Zephyr/Patrick Zephyr Nature Photography

Problems from Calculus: The Analysis of Functions, by Peter D. Taylor (Toronto: Wall & Emerson, Inc., 1992). Reprinted with permission of the publisher.

This book was set in Times Roman by the Consortium using T_EX, Mathematica, and the package A_ST_EX, which was written by Alex Kasman. It was printed and bound by R.R. Donnelley / Kendallville. The cover was printed by R.R. Donnelley.

This book is printed on acid-free paper.

Founded in 1807, John Wiley & Sons, Inc. has been a valued source of knowledge and understanding for more than 200 years, helping people around the world meet their needs and fulfill their aspirations. Our company is built on a foundation of principles that include responsibility to the communities we serve and where we live and work. In 2008, we launched a Corporate Citizenship Initiative, a global effort to address the environmental, social, economic, and ethical challenges we face in our business. Among the issues we are addressing are carbon impact, paper specifications and procurement, ethical conduct within our business and among our vendors, and community and charitable support. For more information, please visit our website: www.wiley.com/go/citizenship.

Copyright ©2017, 2013, 2009, 2005, 2001, and 1998 John Wiley & Sons, Inc. All rights reserved.

No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning or otherwise, except as permitted under Sections 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, 222 Rosewood Drive, Danvers, MA 01923, (508) 750-8400, fax (508) 750-4470. Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, (201) 748-6011, fax (201) 748-6008, E-Mail: PERMREQ@WILEY.COM.

Evaluation copies are provided to qualified academics and professionals for review purposes only, for use in their courses during the next academic year. These copies are licensed and may not be sold or transferred to a third party. Upon completion of the review period, please return the evaluation copy to Wiley. Return instructions and a free of charge return shipping label are available at: www.wiley.com/go/returnlabel. If you have chosen to adopt this textbook for use in your course, please accept this book as your complimentary desk copy. Outside of the United States, please contact your local sales representative.



This material is based upon work supported by the National Science Foundation under Grant No. DUE-9352905. Opinions expressed are those of the authors and not necessarily those of the Foundation.

ISBN-13 978-1-119-37420-6

The inside back cover will contain printing identification and country of origin if omitted from this page. In addition, if the ISBN on the back cover differs from the ISBN on this page, the one on the back cover is correct.

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

PREFACE

Calculus is one of the greatest achievements of the human intellect. Inspired by problems in astronomy, Newton and Leibniz developed the ideas of calculus 300 years ago. Since then, each century has demonstrated the power of calculus to illuminate questions in mathematics, the physical sciences, engineering, and the social and biological sciences.

Calculus has been so successful both because its central theme—change—is pivotal to an analysis of the natural world and because of its extraordinary power to reduce complicated problems to simple procedures. Therein lies the danger in teaching calculus: it is possible to teach the subject as nothing but procedures—thereby losing sight of both the mathematics and of its practical value. This edition of *Calculus* continues our effort to promote courses in which understanding and computation reinforce each other. It reflects the input of users at research universities, four-year colleges, community colleges, and secondary schools, as well as of professionals in partner disciplines such as engineering and the natural and social sciences.

Mathematical Thinking Supported by Theory and Modeling

The first stage in the development of mathematical thinking is the acquisition of a clear intuitive picture of the central ideas. In the next stage, the student learns to reason with the intuitive ideas in plain English. After this foundation has been laid, there is a choice of direction. All students benefit from both theory and modeling, but the balance may differ for different groups. Some students, such as mathematics majors, may prefer more theory, while others may prefer more modeling. For instructors wishing to emphasize the connection between calculus and other fields, the text includes:

- A variety of problems from the **physical sciences** and **engineering**.
- Examples from the **biological sciences** and **economics**.
- Models from the **health sciences** and of **population growth**.

Active Learning: Good Problems

As instructors ourselves, we know that interactive classrooms and well-crafted problems promote student learning. Since its inception, the hallmark of our text has been its innovative and engaging problems. These problems probe student understanding in ways often taken for granted. Praised for their creativity and variety, these problems have had influence far beyond the users of our textbook.

The Seventh Edition continues this tradition. Under our approach, which we call the “Rule of Four,” ideas are presented graphically, numerically, symbolically, and verbally, thereby encouraging students to deepen their understanding. Graphs and tables in this text are assumed to show all necessary information about the functions they represent, including direction of change, local extrema, and discontinuities.

Problems in this text include:

- **Strengthen Your Understanding** problems at the end of every section. These problems ask students to reflect on what they have learned by deciding “What is wrong?” with a statement and to “Give an example” of an idea.
- **ConceptTests** promote active learning in the classroom. These can be used with or without personal response systems (*e.g.*, clickers), and have been shown to dramatically improve student learning. Available in a book or on the web at www.wiley.com/college/hughes-hallett.
- **Class Worksheets** allow instructors to engage students in individual or group class-work. Samples are available in the Instructor’s Manual, and all are on the web at www.wiley.com/college/hughes-hallett.
- **Data and Models**. Many examples and problems throughout the text involve data-driven models.
- **Drill Exercises** build student skill and confidence.